1. Introduction

Wellbore instability is a common problem in oil and gas exploration/production wells during drilling and leads to large expenses. Rock, a natural solid material, is characterised by its anisotropy due to various factors such as the age of formation, lithology, tectonic, deformation, texture and structure, etc. Particularly, weak bedding planes in a rock mass may affect mechanical properties of the rocks and wellbore stability because of its anisotropic strength [1].

Strength is considered as a major parameter to characterise the rock mechanical behaviour. Most of the studies indicate that the strength anisotropy is influenced by dual interaction of orientation of sample bedding plane with respect to the principal stress and the magnitude of confining pressure [2]. Hence, significant efforts have been paid to further understand the anisotropy behaviour in terms of engineering design and analysis to overcome the difficulties in construction of those projects in the anisotropic rock environments. During the last few years, the isotropic model used to determine the anisotropic rock properties revealed a significant uncertainty and unreliability [3 - 5]. In addition, research on wellbore instability was first conducted upon an assumption of linear elastic, isotropic rock, which does not reflect real condition of a rock mass [6].

Several experiments related to the effect of weak bedding planes on rock strength, pore pressure, wellbore instability analysis, etc. have been conducted, suggesting that the bedding plane orientations affected both the elastic constants and yield strength of the rock [4 - 9]. Okland and Cook [10] developed an anisotropic strength theory for wellbore instability problem in the North Sea oil/gas fields, showing that the wellbore instability became worse when drilling parallelly or sub-parallelly to the bedding planes. The plane of weakness theory has been used by Jin et.al. [11] to determine the stability of horizontal wellbores in a naturally fractured formation during well testing. The results showed that the strike of natural fractures could apparently affect the damage form of the sidewall rocks, and a sidewall adjacent to the area of minimum horizontal stress field orientation was the most ready to collapse. In the shale formation, Wu and Tan [12] illustrated that the strength along bedding planes was much weaker than the intact shale material. Effect of the bedding plane failure on wellbore stability in shale was assessed using a transversely isotropic poroelastic and single plane of weakness model. The obtained results showed that the shale bedding planes mainly affected high angle and horizontal wells, which were drilled close
to the minimum horizontal stress direction. Other researchers [13 - 15] found that the analytical process of well drilling and completion in gas shale bearing weak bedding planes depended on logging data, real-time drilling data and in-situ stress tests.

Other elements can also affect wellbore instability when the well encounters weak bedding planes, e.g., the angle between the wellbore and the weak bedding plane. If the angle is high (~ 70° - 90°), the oblique loading on relatively weak laminations likely leads to premature shear failure. Depending on the relative magnitude of the anisotropic rock strength and borehole stress concentration, the breakouts may occur at positions around the borehole. This response is different from those conventionally found in isotropic rock. Due to overburdened diagenesis, shale commonly demonstrates high pore pressure, alignment of phyllosilicates. For this reason, instability of shale is a serious issue, which potentially causes costly problems in many foothills drilling operations, e.g., slip surfaces failing [7, 13]. Thus, a sufficient understanding of the mechanism for instabilities and lost circulation during drilling is needed to reduce the operation cost.

The results of uniaxial compressive, indirect tensile strength and triaxial tests were conducted based on a variety of failure criteria proposed for anisotropic materials. These theories were classified into three groups: mathematical continuous criteria, empirical continuous models, and discontinuous weakness plane theories. The empirical Hoek-Brown failure criterion was fitted to the triaxial data, the corresponding Mohr-Coulomb failure envelope using friction and cohesion parameters. The single plane of weakness theory proposed by Jaeger et.al. [1] is the most widely known. In this theory, the classic Mohr-Coulomb criterion is used to describe the failure of both the bedding planes.

In this paper, we will outline an anisotropic strength model, the effect of weak bedding plane parameters and in-situ stresses on wellbore failure analysis.

2. Modelling

The borehole failure analysis is conducted upon the following assumptions: (i) The rock is heterogeneous and anisotropic; (ii) a set of parallel weak bedding planes exists in which the strengths are low, but the strength of the rock in other directions is uniform; (iii) deformation of rock is low and linear.

2.1. Borehole stress

Before a well is drilled the rock is in a state of equilibrium and the stresses in the Earth under these conditions are known as the far field stresses. Once it is excavated, the static stress state becomes disturbed as the support originally offered by the drilled-out rock is replaced by the hydraulic pressure of the drilling mud and hence causing instability in the rock formation. The disturbed in-situ stress state therefore imposes a different set of stresses in excavation area. Figures 1 and 2 illustrate a schematic distribution of in-situ stresses existing in the formation around a wellbore. The stresses can be resolved into a vertical or overburden stress σ_v, the maximum horizontal in-situ stress σ_H and the minimum horizontal in-situ stress σ_h, which are generally unequal. The direction of well is modelled as shown in Figure 3.

All of the stress components at the wellbore can be calculated in the following steps: (1) Identify the principal in-situ stress state (σ_v, σ_H, σ_h); (2) Transform the stress state (σ_v, σ_H, σ_h) to the stress state (σ_x, σ_y, σ_z) defined with respect to the Cartesian coordinate system attached to the wellbore (Equations 1a - 1f); (3) Find the local stress state (σ_r, σ_θ, σ_a) with respect to the cylindrical coordinated system attached to the wellbore at the distance of a from the center of well, in terms of the stress state (σ_v, σ_H, σ_h) (Equations 2a - 2f); (4) Find the stress state at the wellbore (σ_r, σ_θ, σ_a) (a = r) as Figure 2. For a vertical well, the local stresses can be calculated from the Equations 3a - 3e, as Equations 4a - 4d below:

\[
\sigma_x = \left( \sigma_v \cos^2 a_w + \sigma_h \sin^2 a_w \right) \cos^2 i_w + \sigma_v \sin^2 i_w \quad (1a)
\]
\[
\sigma_y = \sigma_h \sin^2 a_w + \sigma_h \cos^2 a_w \quad (1b)
\]
\[
\sigma_z = \left( \sigma_h \cos^2 a_w + \sigma_h \sin^2 a_w \right) \sin^2 i_w + \sigma_v \cos^2 i_w \quad (1c)
\]
\[
\tau_{xy} = \frac{1}{2} \left( \sigma_H - \sigma_h \right) \sin 2 a_w \cos i_w \quad (1d)
\]
\[
\tau_{xz} = \frac{1}{2} \left( \sigma_H \cos^2 a_w + \sigma_h \sin^2 a_w - \sigma_v \right) \sin 2 i_w \quad (1e)
\]
\[
\tau_{yz} = \frac{1}{2} \left( \sigma_H - \sigma_h \right) \sin 2 a_w \sin i_w \quad (1f)
\]
\[
\sigma_r = \frac{1}{2} \left( \sigma_x + \sigma_y \right) \left( 1 - \frac{a^2}{r^2} \right) + \frac{1}{2} \left( \sigma_x - \sigma_y \right) \quad (2a)
\]
\[
\left( 1 + 3 \frac{a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \cos 2 \theta + \tau_{xy} \left( 1 + 3 \frac{a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \sin 2 \theta + \frac{a^2}{r^2} P_w \quad (2b)
\]
\[
\sigma_t = \frac{1}{2} \left( \sigma_x + \sigma_y \right) \left( 1 + \frac{a^2}{r^2} \right) - \frac{1}{2} \left( \sigma_x - \sigma_y \right) \quad (2b)
\]
\[
\left( 1 + 3 \frac{a^2}{r^2} \right) \cos 2 \theta - \tau_{xy} \left( 1 + 3 \frac{a^2}{r^2} \right) \sin 2 \theta - \frac{a^2}{r^2} P_w
\]
\[
\sigma_a = \sigma_z - 2n(\sigma_x - \sigma_y)\frac{a^2}{r^2} \cos \theta - 4n\tau_{xy}\frac{a^2}{r^2} \sin \theta \quad (2c)
\]

\[
\tau_{\theta z} = (\tau_{xz}\cos \theta - \tau_{xv}\sin \theta)(1 + \frac{\sigma^2}{\tau^2}) \quad (2d)
\]

\[
\tau_{r \theta} = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta (1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}) \quad (2e)
\]

\[
\tau_{rz} = (\tau_{xy}\cos \theta + \tau_{yz}\sin \theta)(1 - \frac{a^2}{r^2}) \quad (2f)
\]

\[
\sigma_r = P_w \quad (3a)
\]

\[
\sigma_z = (\sigma_x + \sigma_y) - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\tau_{xy}\sin 2\theta - P_w \quad (3b)
\]

\[
\sigma_a = \sigma_z - 2n(\sigma_x - \sigma_y)\cos 2\theta - 4n\tau_{xy}\sin 2\theta \quad (3c)
\]

\[
\tau_{\theta z} = 2(\tau_{xy}\cos \theta - \tau_{xv}\sin \theta) \quad (3d)
\]

\[
\tau_{r \theta} = 0; \quad \tau_{rz} = 0 \quad (3e)
\]

\[
\sigma_a = \frac{\sigma_x + \sigma_y}{2} \quad (4a)
\]

\[
\sigma_t = (\sigma_x + \sigma_y) - 2(\sigma_x - \sigma_y)\cos 2\theta - P_w \quad (4b)
\]

\[
\sigma_a = \sigma_z - 2n(\sigma_x - \sigma_y)\cos 2\theta \quad (4c)
\]

\[
\tau_{\theta z} = 0; \quad \tau_{r \theta} = 0; \quad \tau_{rz} = 0 \quad (4d)
\]

where:
- \(\sigma_v\): Vertical (overburden) in-situ stress (Pa, psi);
- \(\sigma_h\): Maximum horizontal in-situ stress (Pa, psi);
- \(\sigma_l\): Minimum horizontal in-situ stress (Pa, psi);
- \(\sigma_x, \sigma_y, \sigma_z\): stress state in the Cartesian coordinate system (Pa, psi);
- \(\tau\): Shear stress (Pa, psi);
- \(\sigma_t\): Tensile stress (Pa, psi);
- \(\sigma_r\): Radius stress (Pa, psi);
- \(P_w\): Wellbore internal pressure (Pa, psi);
- \(a\): Borehole radius (m, in);
- \(r, z, \theta\): Cylindrical co-ordinate system (m, in);

### 2.2. Anisotropic rock strength

According to the rock failure applied to planar anisotropy, which is known as “the single plane of
weakness theory”, the condition for sliding along these planes is given in Equation 5 [1]:

\[ \sigma_1 = \sigma_3 + \frac{2(S_w + \mu_w \sigma_3)}{(1 - \mu_w \cos \beta) \sin 2\beta} \]  

(5)

where:

- \( \sigma_1 \): Maximum principal stress (Pa, psi);
- \( \sigma_3 \): Minimum principal stress (Pa, psi);
- \( S_w \): Inherent shear strength of the planes of weakness (Pa, psi);
- \( \mu_w = \tan \phi_w \): Coefficient of internal friction along weak planes;
- \( \phi_w \): Friction angle of weak plane (degrees);
- \( \beta \): Angle between \( \sigma_1 \) and the normal to the planes of weakness.

Failure will occur in the bulk material based on the same failure criterion as the weak plane such that the maximum principal stress that can be sustained is:

\[ \sigma_3 = 2\tau_0^b \frac{\cos \phi^b}{1 - \sin \phi^b} + \sigma_3 \frac{1 + \sin \phi^b}{1 - \sin \phi^b} \]  

(6)

where:

- \( \tau_0^b \): Cohesion;
- \( \phi^b \): Friction angle of bulk material (degrees).

It is assumed that the bulk material has isotropic strength properties.

3. Case study

The data used in this study are derived from pre-existing works conducted by Crawford et al. [17], with

<table>
<thead>
<tr>
<th>I.D.</th>
<th>Lithology</th>
<th>Source</th>
<th>( \sigma_{\text{triaxial test}} ) (MPa)</th>
<th>Cohesive strength (MPa)</th>
<th>Friction angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lst.</td>
<td>1 Laminated dolomitic limestone</td>
<td>Outcrop McGill &amp; Raney</td>
<td>40</td>
<td>93.9</td>
<td>29.9</td>
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<tr>
<td></td>
<td>2 Green River oil shale (lean)</td>
<td>Mine McLamore &amp; Gray</td>
<td>24</td>
<td>59.6</td>
<td>31.4</td>
</tr>
<tr>
<td></td>
<td>3 Green River oil shale (rich)</td>
<td>Mine McLamore &amp; Gray</td>
<td>21</td>
<td>39.0</td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td>4 Intra-reservoir marl</td>
<td>Cored well</td>
<td>27</td>
<td>30.7</td>
<td>20.2</td>
</tr>
<tr>
<td></td>
<td>5 Intra-reservoir marl</td>
<td>Cored well</td>
<td>28</td>
<td>24.4</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>6 Intra-reservoir marl</td>
<td>Cored well</td>
<td>31</td>
<td>20.7</td>
<td>8.5</td>
</tr>
<tr>
<td>Sst.</td>
<td>7 Fine-grained highly cemented sandstone</td>
<td>Outcrop Chenevert &amp; Gatlin</td>
<td>17</td>
<td>38.4</td>
<td>56.4</td>
</tr>
<tr>
<td></td>
<td>8 Outcrop shale</td>
<td>Outcrop In-house study</td>
<td>35</td>
<td>29.1</td>
<td>29.7</td>
</tr>
<tr>
<td></td>
<td>9 Top seal shale</td>
<td>Cored well</td>
<td>31</td>
<td>18.7</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>10 Top seal shale</td>
<td>Cored well</td>
<td>30</td>
<td>17.6</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>11 Top seal shale</td>
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<td>28</td>
<td>16.4</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>12 Tournemire shale</td>
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<td>25</td>
<td>16.8</td>
<td>21.7</td>
</tr>
<tr>
<td></td>
<td>13 Laminated silty mudstone</td>
<td>Mine Attewell &amp; Farmer</td>
<td>42</td>
<td>14.9</td>
<td>33.8</td>
</tr>
<tr>
<td></td>
<td>14 Bamsley Hards bituminous coal</td>
<td>Mine Pomeroy et al.</td>
<td>28</td>
<td>12.6</td>
<td>36.3</td>
</tr>
</tbody>
</table>

Figure 4. (a) Stress state of rock containing weak bedding planes; (b) rock strength analysis of failure on weak bedding planes and intrinsic rock [16].

Table 1. Summary of 14 database lithologies used in anisotropic shear strength analyses [17]
About 400 triaxial compression testing on 7 individual lithologies (3 top seal shales, 3 intra-reservoir shales and 1 outcrop shale) [17], Narayanasamy et.al. [18], Chris et.al. [19]. These studies used samples collected from Terra Novad, Yasar 2001 of the Upper Miocene - Pliocene Handere formation.

4. Determination of strength anisotropy

Based on the above models and laboratory data, the strength of rock is analysed first. Figure 5 presents a summary of the tests for shale in Table 1. The failure theory is used for interpretation of the test results for the anisotropic rocks, namely the single plane of weakness theory [1]. The failure stress can be computed by specifying cohesion and friction angle (for varying orientation of $\beta$). It is necessary to evaluate two cohesive strength parameters and two coefficients of internal friction for anisotropic materials. As shown in Figure 5, the strength envelope for anisotropic rock is a U-shaped reduction for failure along the weakness plane. At most confining pressures, the criterion overestimates the strength in the regions, where failure is predicted through the intact rock. It is clearly indicated that within the confining pressure varying from 20 MPa to 80 MPa, there is no change in the failure mechanism of the shale. In addition, the results evidently show the anisotropic strength slumped at supreme confining pressure besides, the theoretical method provides advantageous prevision of anisotropy behaviour at higher confining pressures.

5. Cohesive strength model

Cohesive strength or cohesion is the strength of bonding between the particles or surfaces that make up the material. In rock mechanics, the cohesive strength is more specifically the inherent shear strength of a plane across which there is no normal stress. In general, this strength parameter is determined in case of no distinction between failure of a weak plane and failure through the bulk material. In this case, the linearised Mohr failure line can be used, and cohesive strength is estimated by uniaxial tests or triaxial tests. According to experimental
results (Table 1), the cohesive strength data is considered a function of the third-order polynomial of weak plane angle.

The cohesive strength of shale is determined as shown in Figure 6. The continuous variable cohesive strength criterion produces failure envelopes that predict a continuous change. The cohesion theory can be estimated correctly even with $\beta = 0^\circ, 90^\circ$, except at $45^\circ$ and $75^\circ$. By using the polynomial technique, we constate that this technique can be used for predicting the maximum principal stress in function of $\beta$ angle (Figure 7). This result is much better than the single plane of weakness model.

6. Borehole failure

Based on the above models (Equation 5), borehole failure is analysed by applying the data presented in Table 2. The sand production phenomenon is generally taking place through three stages: loss of mechanical integrity of rocks surrounding the borehole, separation of solid particles due to the hydrodynamic force, and transportation of the particles to the surface by production. Excessive sanding or solid production may damage the downhole and surface equipment. Drilling mud is normally chosen in such a way to resist the formation pore pressure, hence preventing the formation fluid from flowing into the wellbore. The drilling mud is not always able to resist the compressive stresses of the wellbore. In this case, shear failures of rock will occur due to the imbalance between stress and rock strength.

In this study, we investigate the possibility of sanding with the presence of weak plane. The returning results allowed us to constate that the field stress regime is normal ($\sigma_v > \sigma_h > \sigma_i$). Figure 8 reveals the possibility of sanding for this case study. As we can see from this figure, the most dangerous case is coded by the red color. In this case, there is a risk zone in the well with the trend angle of weak plane ranging between $60^\circ$ - $90^\circ$, and the changes from $100^\circ$ - $165^\circ$.

However, with any different range of weak plane, the well can be drilled in any azimuth without sanding problem. This result is vital which enables to recommend well plannings, and it is also a good solution for simulation to tackle the risk of drilling well.

7. Conclusions

Results are obtained from various tests applied to the anisotropic strength of shale associated with weak planes, and wellbore failure. This study allows us to draw the following conclusions:

The strength envelope for anisotropic rock shows a U-shaped reduction in strength for failure along the weakness plane.

No change in the failure mechanism of the shale is recorded within the 20 - 80 Mpa interval confining pressure.

The cohesion theory can be estimated correctly even with $\beta = 0^\circ, 90^\circ$, except at $45^\circ$ and $75^\circ$. By using the polynomial technique, it is possible to provide a correctly prediction of the maximum principal stress in function of $\beta$ angle.

---

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Depth (m)</td>
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</tr>
<tr>
<td>Overburden stress (MPa/100 m)</td>
<td>2.4</td>
</tr>
<tr>
<td>Major horizontal stress (MPa/100 m)</td>
<td>2.08</td>
</tr>
<tr>
<td>Minor horizontal stress (MPa/100 m)</td>
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</tr>
<tr>
<td>Pore pressure (MPa)</td>
<td>70</td>
</tr>
<tr>
<td>Cohesion of the weak planes (MPa)</td>
<td>3.5</td>
</tr>
<tr>
<td>Internal friction angle of weak planes ($^\circ$)</td>
<td>14</td>
</tr>
<tr>
<td>Cohesion strength of rock (MPa)</td>
<td>18</td>
</tr>
<tr>
<td>Internal friction angle ($^\circ$)</td>
<td>32</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>Biot’s coefficient</td>
<td>0.9</td>
</tr>
<tr>
<td>Wellbore diameter (mm)</td>
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</tr>
<tr>
<td>Direction of the maximum horizontal principle in-situ stress ($^\circ$)</td>
<td>115</td>
</tr>
<tr>
<td>Fluid density (g/cm$^3$)</td>
<td>1.22</td>
</tr>
</tbody>
</table>

---

Figure 8. Sanding onset in a well of shale formation.
There is a risk zone in the well with the trend angle of weak plane ranging between 60° - 90°, and the azimuth changing from 100° - 165°.

References


